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B.Sc (Part-II) Hons

Langevin's theory of paramagnetism:-

According to Langvin, each atom has permanent magnetic moment m. They only force acting as the atom is due to external field $\vec{B}$. Let θ be the angle of inclination of the axis of the atomic dipole with the direction of the applied field $\vec{B}$. Then the magnetic potential energy of the atomic dipole is

$$U = -mB\cos\theta$$

Now, on classical statics, the number of atom making an angle between θ and $\theta + d\theta$ is

$$dn = Ce^{mB\cos\theta/KT} \sin\theta \, d\theta$$

Where K is Baltzmann's Constant and T is the absolute temprature

putting $mB/KT = \alpha$

then

~~$dn = Ce^{\alpha\cos\theta}$~~

$$dn = Ce^{\alpha\cos\theta} \sin\theta \, d\theta \quad \text{——— (1)}$$

Hence the total number of atomic magnets in unit volume of the

Paramagnetic material

$$n = \int_0^{\pi} dn$$

$$n = \int_0^{\pi} c\, e^{\alpha \cos\theta} . \sin\theta\, d\theta \quad \text{---} \; (2)$$

Let us

$\cos\theta = x$

$\therefore dx = -\sin\theta . d\theta$

for $\theta = 0^\circ$, $\cos 0^\circ = x$, $x = +1$

for $\theta = \pi$, $\cos\pi = x$; $x = -1$

$$\therefore n = -\int_{+1}^{-1} c e^{\alpha x} dx$$

$$= -c \int_{+1}^{-1} e^{\alpha x} dx$$

$$= -c \left[\frac{e^{\alpha x}}{\alpha} \right]_{+1}^{-1}$$

$$n = -c/\alpha \left(e^{-\alpha} - e^{\alpha} \right)$$

$$n\alpha = c\left(e^{\alpha} - e^{-\alpha} \right)$$

$$\therefore c = \frac{n\alpha}{e^{\alpha} - e^{-\alpha}} \quad \text{---} \; (3)$$

The components of each dipole moment parallel to B is $m\cos\theta$. The total ~~mag~~ magnetic moment of all the n atoms contained in unit volume of the gas is the magnetisation M.

$$\therefore M = \int_0^{\pi} m\cos\theta \, dn$$

$$= \int_0^{\pi} m\cos\theta \cdot c e^{\alpha\cos\theta} \cdot \sin\theta d\theta$$

Again let

$$x = \cos\theta$$

$$\therefore dx = -\sin\theta \cdot d\theta$$

$$\therefore M = \int_0^{\pi} m\cos\theta \cdot c e^{\alpha\cos\theta} \cdot \sin\theta \, d\theta$$

$$= -\int_{+1}^{-1} m x \, c e^{\alpha x} dx$$

$$= -cm \int x e^{\alpha x} dx$$

$$= -cm\left[x \int e^{\alpha x} dx - \int\left(\frac{d}{dx}(x)\int e^{\alpha x} dx\right)dx\right]_{+1}^{-1}$$

$$= -cm\left[x \frac{e^{\alpha x}}{\alpha} - \int \frac{e^{\alpha x}}{\alpha} dx\right]_{+1}^{-1}$$

$$= -cm\left[x \cdot \frac{e^{\alpha x}}{\alpha} - \frac{1}{\alpha} \cdot \frac{e^{\alpha x}}{\alpha}\right]_{+1}^{-1}$$

$$= \frac{-cm}{\alpha}\left[-e^{-\alpha} - \frac{e^{-\alpha}}{\alpha} - e^{\alpha} + \frac{e^{\alpha}}{\alpha}\right]$$

$$M = \frac{-cm}{\alpha}\left[\frac{1}{\alpha}(e^{\alpha} - e^{-\alpha}) - (e^{\alpha} + e^{-\alpha})\right]$$

putting the value of c, we get

$$M = \frac{mn}{e^{\alpha}-e^{-\alpha}}\left[(e^{\alpha}+e^{-\alpha}) - \frac{1}{\alpha}(e^{\alpha}-e^{-\alpha})\right]$$

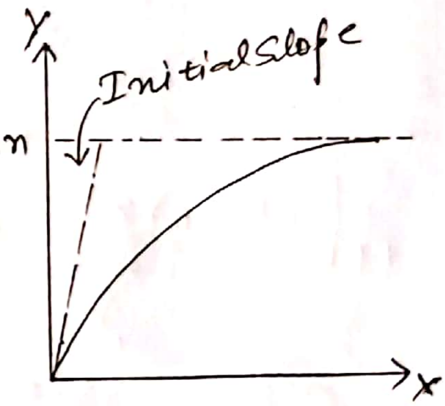
$$\therefore M = mn\left[\frac{e^{\alpha}+e^{-\alpha}}{e^{\alpha}-e^{-\alpha}} - \frac{1}{\alpha}\right]$$

$$M = mn\left[\coth\alpha - \frac{1}{\alpha}\right] \quad \text{--- (4)}$$

or, $M = mn\,L(\alpha)$ ——— (5)

where $L(\alpha) = \left[\coth\alpha - \frac{1}{\alpha}\right]$ is called the Langevin function

The variation of M with α has shown this figure



Case-1 :- At low temprature and large applied field,

$$L(\alpha) \to 1$$

$$\therefore M = mn$$

Thus the saturation point is reached when all the atomic dipoles are parallel to $\vec{B}$.

Case!-II :— Under normal condition α is very small,

then $L(\alpha) = \coth\alpha - \frac{1}{\alpha} = \frac{\alpha}{3}$

$$\therefore M = \frac{mn\alpha}{3}$$

$$= \frac{mn^2 B}{3KT}$$

$$= \frac{mn^2 \mu_0 H}{3KT} \quad (\because B = \mu_0 H)$$

$$\therefore \chi = \frac{M}{H}$$

$$= \frac{mn^2 \mu_0 H}{3KT \cdot H}$$

$$= \frac{mn^2 \mu_0}{3KT}$$

$$= \frac{\mu_0 m^2 n}{3K} \cdot \left(\frac{1}{T}\right)$$

$$= c\left(\frac{1}{T}\right)$$ (where $c = \mu_0 m^2 n / 3K$ = Constant

This Constant is Called curie Constant

$$\therefore \chi = \frac{c}{T} \quad \text{—— (6)}$$

$\therefore \boxed{\chi \propto \frac{1}{T}}$

According to weiss' modification

$$\chi = \frac{c}{T - \theta}$$

where θ is Called curie temprature, which is a characteristic of the Substance.